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Research Note

Calculation of Sound Transmission Loss in Ocean Using Different Wave Equation Models

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Abstract - Underwater acoustics is the study of the propagation of sound in water and the interaction of the acoustic (sound) waves with the water, its contents, and its boundaries. While sound moves at a much faster speed in the water than in air, the distance that sound waves travel is primarily dependent upon the sound speed profile of the ocean. The need to model and study acoustic propagation in the sea has always been in demand. Practical issues with forecasting sonar performance in support of anti-submarine warfare (ASW) operations during World War II led to the earliest attempts at modeling sound propagation in the sea. The theoretical basis underlying all mathematical models of acoustic propagation is the wave equation. The wave equation itself is derived from the more fundamental equations of state, continuity, and motion. As a mathematical expression of acoustic physical properties, a numerical acoustic field can describe the physical laws of ocean acoustic propagation with simple and clear numerical solutions. Commonly used computational ocean acoustic theories include the parabolic equation (PE) model, normal modes, the wavenumber integration method and the ray model. This paper aims at studying all these models briefly and focusing on the PE models as they are considered to be fast and flexible for range-dependent acoustic propagation problems like those of the Indian Ocean Region (IOR). Additionally, it compares the three models of PE equations and states that the accuracy, speed, and efficiency of any model depend on the sound speed profiles. The findings include that there has not been a single model that guarantees to work efficiently in all places and situations. Rather, different models work perfectly with different scenarios, making them compatible with different profiles. The PE-RAM model, although the most widely used model, is shown to work poorly when it comes to smooth sound profiles. On the other hand, it proved to be the fastest for rough profiles.

Keywords - Underwater Acoustics, Ray theory, Normal model, Multipath expansion, Fast Field, Parabolic equation, RAM, CCPME, CSMPE

Introduction- The ocean contains an abundance of energy, minerals, and biological resources. The urgent requirements of marine research and development have posed new challenges for the detection, identification, positioning, and communication of underwater targets. At present, sound waves are the main means for remotely transmitting information underwater; therefore, it is of great practical significance to thoroughly study and understand the laws of underwater acoustic propagation. Commonly used computational ocean acoustic theories include the parabolic equation (PE) model, normal modes, the wavenumber integration method, and the ray model (Tu et al., 2020).

Development of the wave propagation models: Formulations of acoustic propagation models (APMs) generally begin with the three-dimensional, time-dependent wave equation. For most applications, a simplified linear, hyperbolic, second-order, time-dependent partial differential equation is used:

$$\nabla^2\Phi = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} \qquad (4.1)$$

where ∇2 is the Laplacian operator [= (∂2/∂x2) + (∂2/∂y2) + (∂2/∂z2)], Φ is the potential function, c is the speed of sound, and t is the time. Subsequent simplifications incorporate a harmonic (single-frequency, continuous wave) solution in order to obtain the time-independent Helmholtz equation.Specifically, a harmonic solution is assumed for the potential function Φ:

$$\Phi = \varphi\, e^{-i\omega t}$$

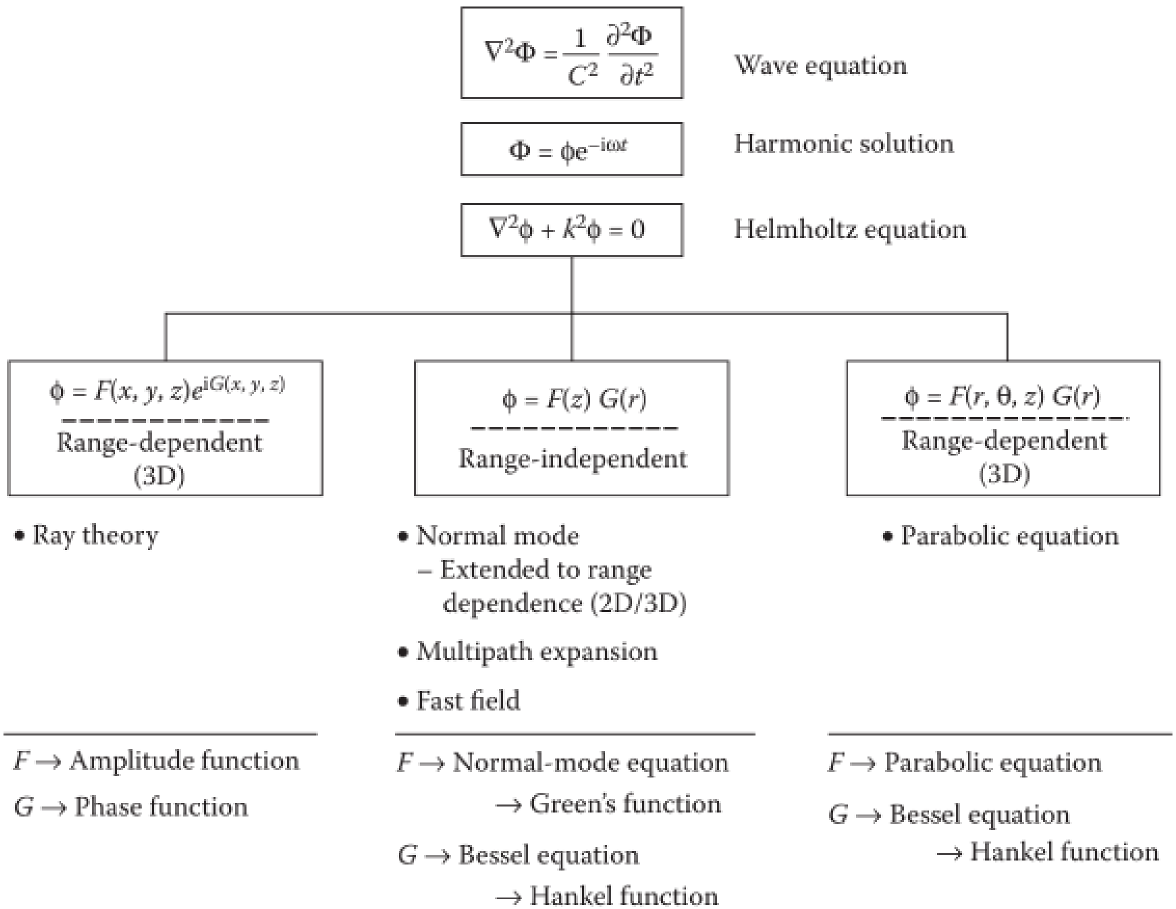
where ϕ is the time-independent potential function, ω is the source frequency (2πf), and f is the

$$\nabla^2\phi + k^2\phi = 0 \qquad (4.3a)$$

acoustic frequency. Then the wave equation 4.1 reduces to the Helmholtz equation where k = ω/c = 2π/λ is the wavenumber and λ is the wavelength. In cylindrical coordinates, Equation 4.3a becomes

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} + k^2(z)\phi = 0 \qquad (4.3b)$$

Equation 4.3a is referred to as the time-independent (or frequency-domain) wave equation. Equation 4.3b, in cylindrical coordinates, is commonly referred to as the elliptic-reduced wave equation. Various theoretical approaches are applicable to the Helmholtz equation. The approach used depends upon the specific geometrical assumptions made for the environment and the type of solution chosen for ϕ.



There are 5 canonical solutions/modeling techniques which are namely - ray theory, normal mode, multipath expansion, fast field, and parabolic equation (PE) techniques. Within these five categories, a further subdivision can be made according to range independent and -dependent models. Range independence means that the model assumes a horizontally stratified ocean in which properties vary only as a function of depth. Range dependence indicates that some properties of the ocean medium are allowed to vary as a function of range (r) and azimuth (θ) from the receiver, in addition to depth (z) dependence. Such range-varying properties commonly include sound speed and bathymetry, although other parameters such as sea state, absorption and bottom composition may also vary (Etter, 2018). The figure above clearly categorizes these models and explain the function that each one of them uses.

Applications

a. **Ray Theory Model:** Ray-theoretical models calculate wave equation on the basis of ray tracing. It is used to to predict noise emissions from isolated wind turbines and wind parks(*Prospathopoulos & Voutsinas, 2007*), modeling and imaging of seismic data (*Iversen et al., n.d.*)[4], modeling of the ultra-wideband on body radio channel(*UWB On-body Radio Channel Modeling Using Ray Theory and Subband FDTD Method, n.d.*). A ray theory approach to investigate the influence of flow velocity profiles on transit times in ultrasonic flow meters for gas and liquid[6]. This model is developed based on the geometrical acoustics approximation. The geometrical acoustics approximation is a condition in which the fractional change in the sound-speed gradient over a wavelength is small compared to the gradient c/λ, where c is the speed of sound and λ is the acoustic wavelength. Specifically,

$$\frac{1}{A}\nabla^2 A \ll k^2$$

In other words, the sound speed must not change much over one wavelength.The geometrical acoustics approximation effectively limits the ray-theoretical approach to the high-frequency domain. With appropriate frequency-dependent (diffraction) corrections and proper evaluation of caustics, ray theory can be extended to frequencies lower than those normally associated with the geometrical acoustics approximation. Under these conditions, the approach is commonly termed “ray theory with corrections.” Another drawback is propagation models based on ray-tracing techniques generally treat bottom reflection as specular and reduce the intensity through application of a bottom reflection loss. However, acoustic energy can be transmitted into the bottom where it is subsequently refracted, attenuated, and even transmitted back into the water column at some distance down range(*Etter, 2018*).

b. **Normal Model:** This models has varied applications like expansion to AE Waves in Finite Plates(*Gorman & Prosser, 2013*), Dynamics of Gravity Oscillations in a Shallow Water Model, solving Differential Equations with Neural Networks*(Solving Differential Equations With Neural Networks: Application to the Normal-mode Equation of Sound Field Under the Condition of Ideal Shallow Water Waveguide, n.d.)*.To solve the equation using this particular model, cylindrical symmetry is assumed in a stratified medium (i.e., the environment changes as a function of depth only). One advantage of normal-mode solutions over ray-theoretical methods is that TL can easily be calculated for any given combination of frequency and source depth (z0) at all receiver depths (z) and ranges (r). Ray models, on the other hand, must be executed sequentially for each change in source or receiver depth. A disadvantage associated with normal-mode solutions is the degree of information required concerning the structure of the sea floor. In order to execute effectively, this type of model generally requires knowledge of the density as well as the shear and compressional sound speeds within the various sediment layers. Unlike ray-theoretical solutions, wave-theoretical solutions inherently treat dispersion effects. Dispersion is the condition in which the phase velocity is a function of the acoustic frequency If present, dispersion effects are most noticeable at low frequencies. Normal mode approaches tend to be limited to acoustic frequencies below 500 Hz due to computational considerations.

c. **Multipath Expansion:** Multipath expansion techniques expand the acoustic field integral representation of the wave equation in terms of an infinite set of integrals, each of which is associated with a particular ray-path family. This method is sometimes referred to as the “WKB method” since a generalized WKB (Wentzel–Kramers–Brillouin) approximation is used to solve the depth-dependent equation derived from the normal mode solution. Each normal mode can then be associated with corresponding rays. Unlike ray-theoretical solutions, however the WKB method normally accounts for first-order diffraction effects and caustics. This approach is particularly applicable to the modeling of acoustic propagation in deep water at intermediate and high frequencies. Multipath expansion models thus have certain characteristics in common with ray models.(*Etter, 2018*).

d. **Fast Field:** It is used in underwater acoustics and seismology. In underwater acoustics, fast-field theory is also referred to as “wavenumber integration.” In seismology, this approach is commonly referred to as the “reflectivity method” or “discrete-wavenumber method.” Historically, models based on fast-field theory did not allow for environmental range dependence(*Etter, 2018*).

e. **Parabolic Equation:** The parabolic equation method is widely used in ocean acoustics and seismology. The parabolic approximation method was successfully applied to model tropospheric radiowave propagation over land in the presence of range-dependent refractivity *(A Terrain Parabolic Equation Model for Propagation in the Troposphere, n.d.*), X-ray diffraction optics*(Application of the Parabolic Wave Equation to X-ray Diffraction Optics, 1999)*, microwave waveguides, laser beam propagation, plasma physics, and seismic wave propagation.The PE (or parabolic approximation) approach replaces the elliptic-reduced equation with a PE. The PE is derived by assuming that energy propagates at speeds close to a reference speed—either the shear speed or the compressional speed, as appropriate. The computational advantage of the parabolic approximation lies in the fact that a parabolic differential equation can be marched in the range dimension, whereas the elliptic-reduced wave equation must be numerically solved in the entire range-depth region simultaneously. At the time Tappert introduced the PE method to the underwater acoustics community, there was a critical need for a capability to predict long-range, low-frequency sound propagation, as would occur in the vicinity of the sound channel axis. Since this type of propagation is characterized by low-angle, nonboundary interacting energy, the PE method was ideally suited to this

purpose.In general, PE models propagate the acoustic field only in the forward direction, thus excluding backscatter. Among all these mentioned models, the PE model has the advantage of being fast and flexible when solving range-dependent acoustic propagation problems.*(Tu et al., 2020)*.
Keeping the current scenarios in mind, it has been found that IOR is deep water and we need to use it for the low frequency sound propagation. So, the best model type was the Parabolic equation.

Model type	Applications							
	Shallow water				Deep water			
	Low frequency		High frequency		Low frequency		High frequency	
	RI	RD	RI	RD	RI	RD	RI	RD
Ray theory	○	○	◑	●	◑	◑	●	●
Normal mode	●	◑	●	◑	●	◑	◑	○
Multipath expansion	○	○	◑	◑	◑	◑	●	◑
Fast field	●	◑	●	◑	●	◑	◑	◑
Parabolic equation	◑	●	○	○	◑	●	◑	◑

Low frequency (<500 Hz)
High frequency (>500 Hz)

RI: Range-independent environment
RD: Range-dependent environment

● Modeling approach is both applicable (physically) and practical (computationally)

◑ Limitations in accuracy or in speed of execution

○ Neither applicable nor practical

Parabolic Equation in detail: Research on underwater acoustic propagation modeling theory began in the 1960s. Initially, only ray theory and the horizontally layered normal mode theory existed. Their ability to deal with problems was limited, and they could only calculate range-independent problems. Numerous contributions have been made in the enhancement of the Parabolic Equation (PE) approximation method, which has been shown to be a useful tool for solving realistic problems in many different scientific fields. Evidence of its usefulness is the application of PE to solve ocean acoustic propagation problems. In the 1970s, Hardin and Tappert introduced the PE method in the field of underwater acoustics for the first time and approximated the Helmholtz equation as a two-dimensional equation that was related only to the range and depth and was independent of the azimuth. In the 1980s, Davis et al. derived a generalized PE model using the operator method; the derivation based on a series expansion of the square-root operator Q enables the formulation of better PE approximations with a wide-angle capability. Greene5 and Claerbout6 selected different coefficients and derived their respective wide-angle PE models. Accordingly, interest in PE techniques has steadily grown within the ocean acoustic modeling community. Based on the idea of parabolic approximation, many parabolic model schemes have been proposed. In scientific computing and numerical simulations in engineering, the spectral method (SM), FDM and finite element method are the three major discrete numerical methods.*(Tu et al., 2020)*.

Different types of PE model program:

- **RAM (Range Acoustic Model):** Collins was the first to implement the wide-angle PE numerical solution based on the high-order Pad´e approximation and expanded the propagation angle to nearly 90◦. This process solved many practical problems, such as the self-starter to obtain an initial condition, "split-step" high-order Pad´e series approximation, energy loss problem caused by a step approximation and treatment of an inclined seafloor boundary. Then, the classic underwater range-dependent acoustic model (RAM) program was developed, and the depth operator was discretized using the finite difference method (FDM). This approach of replacing the depth operator with a tridiagonal matrix can address piecewise continuous depth variations in acoustic parameters. After discretizing in the depth direction, the numerical solution involves repeatedly solving tridiagonal systems of equations.*(Tu et al., 2020)*.

- **CSMPE(Chebyshev–Tau Spectral Method) :** The (Spectral Methods)SM to solve the PE, originates from the method of weighted residuals. It uses orthogonal polynomials (triangular polynomials, Chebyshev polynomials, Legendre polynomials) as the basis functions and applies finite-term series to approximate the variables to be solved. The greatest advantage of the SM is that it exhibits exponential convergence; i.e. when the solution of the original equation is sufficiently smooth, the approximate solution obtained by the SM will quickly converge to the exact solution.the SM is frequently used in various mathematical and physical problems, such as computational fluid dynamics, chemical measurements and electricity.Solving an acoustic wave equation using a parabolic approximation is a popular approach for many existing ocean acoustic models. Considering the idea and theory of the wide-angle rational approximation, a discrete PE model using the Chebyshev spectral method (CSM) is derived, and the code is developed. Most recently, Tu et al. implemented a Chebyshev–Tau SM to solve acoustic normal modes with a stratified marine environment. In applying the SM to solve the PE model, Tu et al. presented the standard PE model using the Chebyshev spectral method (CSM) to process a single layer of a body of water with constant density and no attenuation.This method is currently suitable only for range-independent waveguides.Compared with the RAM results, the CSMPE results have higher accuracy in the simple case of constant sound speed. However, the CSM has a longer runtime than the RAM despite having fewer discrete points. Thus, the CSMPE is slower than the RAM. Even considering that the RAM is a well-optimized code, the speed of the CSMPE is not satisfactory, which is its main disadvantage.*(Tu et al., 2020)*. The CSM for the discrete PE model is feasible and reliable, the results are credible, and the CSM has higher accuracy than the classic PE model (the RAM program based on the FDM) in a range-independent environment. The disadvantage of the CSM is the large amount of calculations involved; in the calculation of the CSMPE program, it is necessary to solve the dense matrix equations multiple times, while the FDM must solve only the larger-scale tridiagonal matrix algebraic equations. In addition, the CSMPE program is used only in simple marine environments above a flat, horizontal ocean floor; that is, variations in the sound speed profile with range are not considered.

- **CCMPE (Chebyshev Collocation Method) :** The collocation method is a kind of spectral method that is based on the principle of weighted residual minimization. The CCM provides a program for computing the sound pressure field when the sea surface and bottom are range independent. This method first interpolates the acquired data of the sound speed, density and attenuation profiles to the CGL points. After modifying the depth operator matrix with the boundary conditions, complex matrix algebraic equations for solving the pressure field are formed that can be solved by applying numerical libraries and algorithms. In general, CCMPE uses fewer discrete points to match or exceed the RAM's accuracy, especially when the acoustical profiles are smooth like an ideal fluid waveguide with a constant sound speed profile, an ideal fluid waveguide with a munk sound speed profile. However, in cases where the sound speed, density and attenuation profiles are not sufficiently smooth like an ideal fluid waveguide with a barents sea sound speed profile, an ideal fluid waveguide with surface duct sound speed profile the CCM should use more CGL points to obtain convincing results.The calculation speed of CCMPE is much faster than that of CSMPE but the computational time is longer than that of the RAM.

Limitations: This work mainly focuses on the three models of solving parabolic equations, but there are many other models as well. Moreover, these five canonical models mentioned in the work are the older versions. Recent developments have been made in all these models that have made significant progress in improving these models. Additionally, this work mainly pays attention to the rough and smooth sound profiles, limiting the research.

Way forward: Keeping the limitations in mind, detailed research can be done on the five mentioned above models with their developed versions so as to incorporate the new updates.

Additionally, there can be a further study comprising all the different models of solving the parabolic equations.

Conclusion: The vast ocean holds significant potential in terms of energy, minerals, and biological resources. However, effectively harnessing these resources requires advancements in marine research and development. One of the key challenges in this domain involves detecting, identifying, positioning, and communicating with underwater targets. Currently, sound waves serve as the primary method for transmitting information underwater. Therefore, comprehensively studying and comprehending the principles governing the propagation of sound in water is of immense practical importance. Several computational models are commonly employed to understand underwater acoustic propagation. These models include the parabolic equation (PE) model, normal modes, the wavenumber integration method, and the ray model. Each of these approaches provides a unique perspective on the behavior of sound waves in water and enables researchers to tackle different aspects of underwater acoustics. The parabolic equation model, for instance, is a widely utilized computational technique that approximates the behavior of sound waves by solving a simplified version of the wave equation. Normal modes, on the other hand, describe the natural oscillations of sound in an ocean waveguide and aid in analyzing the transmission of sound over long distances. The wavenumber integration method involves integrating the contributions of various wave components to determine the overall sound field. Lastly, the ray model simplifies the acoustic propagation process by approximating sound waves as rays that travel in straight lines, thereby facilitating quick estimations of sound paths and arrival times. By employing these computational models, scientists and researchers can gain valuable insights into underwater acoustics, which in turn can inform the development of efficient systems for underwater communication, target detection, and resource exploration. Understanding the intricacies of sound propagation in water opens up new avenues for sustainable and responsible utilization of the ocean's vast resources.

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